

A Theory of Network Games Part II: Potentials and Anti-Potentials

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Abstract

Building on our work from part I, we provide revealed preference conditions under which a network game has a potential. Our analysis divides into two parts. First, the novel *reciprocal prism condition* characterizes when the interaction between two players, hold others' actions fixed, is one of pure common interest or pure conflict. We then show that, to verify whether the game as a whole has a potential, it suffices to check whether utility scales are consistent along chordless cycles in the interaction network.

1 Introduction

Potentials are an important tool in the literature on network games. Many network games, particularly those with symmetric interactions, have a potential structure, which can greatly simplify equilibrium analysis (e.g. Bramoullé et al., 2014; Babichenko and Tamuz, 2016; Zenou and Zhou, 2023). Moreover, having a potential inspires confidence in model predictions because, even in a large and complex network, players can readily learn how to play equilibrium strategies from feedback on neighbors' behavior. Having addressed properties that ensure additively separable utility representations in part I, we turn our attention in part II towards revealed preference conditions under which a network game admits a potential.

One might expect that useful characterizations, based on more general findings for potential games, already exist. Sadly, such expectations are mistaken. The famous four-cycle condition of Monderer and Shapley (1996) is inextricably linked to cardinal utility, while the ordinal test of Voorneveld and Norde (1997) requires checking arbitrarily long sequences of strategy profiles to rule out improvement cycles. We provide novel examples demonstrating that, in general games, improving on the latter characterization is impossible, but network

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games are special. The independence conditions from part I, which capture the idea that strategic interactions are bilateral, lead to a cardinal utility representation that we can identify directly from preference comparisons. Using this, we can bound the length of cycles we need to check to ensure that a potential exists.

Building on our work from part I, we start from utilities that are additively separable across opponents: assume we can represent preferences via

$$u_i(\mathbf{s}) = \sum_{j \neq i} g_{ij}(s_i, s_j) \quad (1)$$

for each player i , in which s_i denotes the action of player i . We break our problem into two steps. First, we identify revealed preference conditions under which the bilateral terms g_{ij} and g_{ji} are affine transformations of one another. The property, which we call the **reciprocal prism condition**, implicitly imposes a restriction on *cross-differences*: if we have

$$\begin{aligned} g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) - (g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j)) \\ = g_{ij}(\tilde{s}'_i, \tilde{s}'_j) - g_{ij}(\tilde{s}_i, \tilde{s}'_j) - (g_{ij}(\tilde{s}'_i, \tilde{s}_j) - g_{ij}(\tilde{s}_i, \tilde{s}_j)) \end{aligned}$$

for player i , then also

$$\begin{aligned} g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i) - (g_{ji}(s'_j, s_i) - g_{ji}(s_j, s_i)) \\ = g_{ji}(\tilde{s}'_j, \tilde{s}'_i) - g_{ji}(\tilde{s}_j, \tilde{s}'_i) - (g_{ji}(\tilde{s}'_j, \tilde{s}_i) - g_{ji}(\tilde{s}_j, \tilde{s}_i)) \end{aligned}$$

for player j . If this holds for every pair of players, there is a canonical utility

$$u_i(\mathbf{s}) = v_i(s_i) + \sum_{j \neq i} (g_{ij}(s_i, s_j) + w_{ij}(s_j))$$

for each player i such that $g_{ij}(s_i, s_j) = \gamma_{ij}g_{ji}(s_j, s_i)$ for some constants γ_{ij} . We identify conditions under which the associated constant is positive, and in this case we say there is a **bilateral potential** for players i and j —the bilateral strategic interaction is one of common interest. As a byproduct of our analysis, we also obtain conditions under which the constant is negative, allowing us to identify games with pure bilateral conflict.

Assuming each bilateral interaction is one of common interest, the second step is to check whether we can simultaneously rescale the utilities of all players so that $g_{ij}(s_i, s_j) = g_{ji}(s_j, s_i)$ for all pairs. We present an example showing that even this step alone may require checking for long improvement cycles, but it also hints at conditions under which checking shorter cycles is sufficient. A network game with bilateral potentials implicitly defines a strategic interaction graph in which players i and j are linked if the corresponding bilateral potential is not identically zero. The key difficulty is the existence of long cycles in this graph, but a cycle only matters if it cannot be cut short along a chord. The length of the longest such cycle bounds the length of improvement cycles one must check. In particular, if the interaction graph is a tree, the game always admits a potential, and if the interaction graph is chordal,¹ then checking 3-cycles is sufficient.

¹This means that any cycle of length 4 or greater contains a chord, a link connecting two players who are not adjacent in the cycle.

Though we focus on games in which a network mediates strategic interactions, our results can apply beyond this context. Our conditions for having a bilateral potential require that each of the two players has another opponent whose actions can offset changes in the partner’s action. While this prevents direct application to two player games, there is nothing in the formalism that requires an *opponent* to be the offsetting force. If each of two players has joint preferences over outcomes in the game, and over some other set of outcomes that are independent of the game, then we can derive a cardinal scale from revealed preference and check whether the game has a bilateral potential given this representation.

Our findings contribute to our collective understanding of network games as well as the broader literature on potential games. We identify interpretable revealed preference conditions guaranteeing that a game has a potential, and we highlight precisely what structural features of the interaction network make it harder or easier to determine whether a potential exists. Moreover, to the best of our knowledge, we provide the first revealed preference test for a potential that comes with a bound on the length of cycles one must check.

Related Work

While part I (Root and Sadler, 2026) directly addresses the simple utility functions used in the vast majority of papers on network games (e.g. Ballester et al., 2006; Bramoullé and Kranton, 2007; Belhaj et al., 2014; Galeotti et al., 2020; Bervoets and Makihara, 2025), part II relates more specifically to the subset of this literature that leverages tools for potential games (Bramoullé et al., 2014; Babichenko and Tamuz, 2016; Parise and Ozdaglar, 2019; Zenou and Zhou, 2023). Rather than assuming a particular utility with a potential, we identify revealed preference conditions under which a network game must have a potential.

Babichenko and Tamuz (2016) and Bayer et al. (2023) offer the two most closely related analyses. Babichenko and Tamuz (2016) define a graph associated with an arbitrary game—a link exists between i and j if the two players are payoff relevant to each other—and highlight a special relationship between this graph and an exact potential for the game, when the latter exists. Their analysis shows that, when a game has an exact potential, we can always write this potential as a sum of terms that depend on the actions of players in maximal cliques in the payoff-relevance graph. In the special case of a bipartite payoff-relevance graph, these maximal cliques necessarily each have size 2, and their results then imply that players’ payoffs are additively separable across opponents, and the potential has the same canonical form we identify.

Bayer et al. (2023) start from a canonical network game with linear best replies and identify conditions on the network weights ensuring that a potential exists. Restricted to this special class of network games, their findings anticipate part of our Theorem 2, highlight conditions under which a potential exists given that bilateral potentials exist. We improve on this finding in two ways. First, we extend to a more general class of games. Second, we show that their “transitive relative importance” condition need only be checked for chordless cycles in the network—we need not check all cycles.

2 Network Games and Potentials

Consider a normal form game $(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$ —each player $i \in N$ has available actions S_i and earns a payoff $u_i(\mathbf{s})$ from the profile $\mathbf{s}$. The game has an **exact potential** if there exists a function $\Phi : S \rightarrow \mathbb{R}$ such that

$$\Phi(s'_i, s_{-i}) - \Phi(s_i, s_{-i}) = u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i})$$

for every player i , every opponent profile $s_{-i} \in S_{-i}$, and every pair of actions $s_i, s'_i \in S_i$. The game has an **ordinal potential** if there exists a function $\Phi : S \rightarrow \mathbb{R}$ such that

$$\Phi(s'_i, s_{-i}) \geq \Phi(s_i, s_{-i}) \iff u_i(s'_i, s_{-i}) \geq u_i(s_i, s_{-i})$$

for every player i , every opponent profile $s_{-i} \in S_{-i}$, and every pair of actions $s_i, s'_i \in S_i$.

Many network games have potentials, and our goal is to understand what this substantively implies about players' preferences. We build on our results from part I that identify independence conditions under which we can represent player preferences using additively separable utilities:

$$u_i(\mathbf{s}) = \sum_{j \neq i} g_{ij}(s_i, s_j). \quad (2)$$

Our main findings in part II highlight revealed preference conditions implying that, with an appropriate rescaling, we have

$$g_{ij}(s_i, s_j) = g_{ji}(s_i, s_j) + v_{ij}(s_i) + w_{ij}(s_j)$$

for some functions v_{ij} and w_{ij} . If we can rescale every player's utility so that the above condition holds simultaneously for all pairs, then the rescaled game has an exact potential

$$\Phi(\mathbf{s}) = \sum_{i=1}^n v_i(s_i) + \sum_{i=1}^n \sum_{j < i} g_{ij}(s_i, s_j)$$

in which $v_i(s_i) = \sum_{j < i} v_{ij}(s_i)$. We subsequently derive conditions on the interaction network that ensure a global potential exists. As a byproduct of this analysis, we also obtain conditions under which bilateral interactions represent pure conflict, with $g_{ij}(s_i, s_j) = -g_{ji}(s_j, s_i)$ after translation and rescaling.

As in part I, we maintain several technical assumptions throughout the paper. The action sets S_i are path-connected topological spaces, and preferences are continuous. We call an opponent j is **essential** for player i if there exists an action $s_i \in S_i$, two actions s_j, s'_j for opponent j , and a profile of actions $s_{-ij} \in S_{-ij}$ for the other opponents such that i has a strict preference:

$$(s_i, s'_j, s_{-ij}) \succ_i (s_i, s_j, s_{-ij}).$$

We assume that all players have at least two essential opponents. We also take as given that every player's preferences have an additively separable representation (2). From part I, this is ensured if action spaces are rich enough and preferences satisfy **joint independence**:

for any actions $s_i, s'_i \in S_i$, any two opponents j and k , and any opponent action profiles $s_{-i}, s'_{-i}, t_{-i}, t'_{-i} \in S_{-i}$, we have

$$\begin{aligned} (s_i, s_{-i}) \succeq_i (s'_i, s'_{-i}), \quad (s'_i, t'_k, s'_{-ik}) \succeq_0 (s_i, t_j, s_{-ij}), \quad (s'_i, s'_k, t'_{-ik}) \succeq_0 (s_i, s_j, t_{-ij}) \\ \implies (s'_i, t'_{-i}) \succeq_0 (s_i, t_{-i}), \end{aligned}$$

with the last comparison being strict if any one of the first three is strict.

Why Checking for a Potential is Hard

In general games, no practical revealed preference condition can guarantee that a potential exists. The famous four-cycle condition of Monderer and Shapley (1996) depends critically on *cardinal preferences*: a game has an exact potential if and only if

$$\begin{aligned} u_i(s'_i, s_j, s_{-ij}) - u_i(s_i, s_j, s_{-ij}) + u_j(s'_i, s'_j, s_{-ij}) - u_j(s'_i, s_j, s_{-ij}) \\ + u_i(s_i, s'_j, s_{-ij}) - u_i(s'_i, s'_j, s_{-ij}) + u_j(s_i, s_j, s_{-ij}) - u_j(s_i, s'_j, s_{-ij}) = 0. \end{aligned}$$

Using only ordinal comparisons, the best characterization we have comes from Voorneveld and Norde (1997), which requires ruling out weak improvement cycles of arbitrary length. A **weak improvement cycle** is a sequence of strategy profiles $s^0, s^1, \dots, s^m = s^0$ such that for each $k = 0, 1, \dots, m-1$, there exists a player i such that $s^k_{-i} = s^{k+1}_{-i}$, and for this player i we have $u_i(s^{k+1}) \geq u_i(s^k)$, with at least one strict inequality in the sequence.² Checking long cycles of strategy profiles is onerous, and this characterization has accordingly seen little application in practice. While Voorneveld and Norde (1997) show that checking cycles of length 4 is not enough—their example has a cycle of length 6, but none of length 4 or less—one might still hope for a reasonable bound on the length of the shortest cycle. Unfortunately, as the following original example demonstrates, even otherwise well behaved games may require checking arbitrarily long cycles.

Suppose there are two players, and action sets are $S_1 = S_2 = \{0, 1, 2, \dots, m\}$ for some odd integer m . Define the functions

$$\begin{aligned} x(s_2) &= \begin{cases} 0 & \text{if } s_2 = 0 \\ s_2 - \epsilon \cdot (-1)^{s_2} & \text{if } 0 < s_2 < m \\ m & \text{if } s_2 = m \end{cases} \\ y(s_1) &= \begin{cases} \frac{1+\epsilon}{2} & \text{if } s_1 = 0 \\ s_1 + \epsilon \cdot (-1)^{s_1} & \text{if } 0 < s_1 < m \\ m - \frac{1+\epsilon}{2} & \text{if } s_1 = m \end{cases} \end{aligned}$$

for some very small $\epsilon > 0$. Player utilities are

$$u_1(s_1, s_2) = -(s_1 - x(s_2))^2, \quad u_2(s_1, s_2) = -(s_2 - y(s_1))^2.$$

²Ruling out weak improvement cycles is enough to obtain an order $\succeq$ on S that functions as a potential, but ensuring that this order has a real representation requires further technical assumptions.

In this game, each player improves her payoff by moving closer to a particular function of her opponent's action. Since these functions are strictly increasing, payoffs feature strictly increasing differences, and the game exhibits strategic complements. Note that preferences are also strictly single-peaked.

Despite all of its helpful structure, this game requires us to check long cycles to verify that it cannot have a potential. Here is the shortest (and only) improvement cycle, which has length $2(m + 1)$:

$$(0, 0) \rightarrow (0, 1) \rightarrow (2, 1) \rightarrow (2, 3) \rightarrow \dots \rightarrow (m - 1, m) \rightarrow (m, m) \rightarrow (m, m - 1) \\ \rightarrow (m - 2, m - 1) \rightarrow (m - 2, m - 3) \rightarrow \dots \rightarrow (1, 0) \rightarrow (0, 0).$$

To see why no shorter cycle exists, consider the distance $|s_1 - s_2|$. There are only two improving changes that can increase this distance: the change from $(0, 0)$ to $(0, 1)$ and the change from (m, m) to $(m, m - 1)$. Every other step in an improvement path must weakly decrease the distance $|s_1 - s_2|$. Using this, we see that a cycle cannot include (s_1, s_2) with $|s_1 - s_2| \geq 2$. If such a cycle existed, we would have $|s_1 - s_2| = c \geq 2$ for every (s_1, s_2) that appears, which means that every move in the cycle involves a player i switching from $s_{-i} - c$ to $s_{-i} + c$ or vice versa. Any such improvement path cannot turn around when it reaches the highest or lowest possible actions. This means that, within an improvement cycle, we have $|s_1 - s_2| \leq 1$ for every profile (s_1, s_2) that appears. The cycle exhibited above includes all such profiles that are not equilibria, and the improvements that do not lead to equilibria are exactly those we have used.

Network games present a special case in which we can overcome the difficulty illustrated in this example. When player preferences admit an additively separable representation

$$u_i(\mathbf{s}) = \sum_{j \neq i} g_{ij}(s_i, s_j),$$

with at least two essential opponents, such a representation is *cardinal*—that is, unique up to a positive affine transformation. Intuitively, separability allows us to use the utility change from shifting one neighbor's action as a measuring stick for shifts in other neighbors' actions. Once we have a cardinal scale, we can implicitly invoke Monderer and Shapley (1996). Our main challenge is to identify conditions ensuring that $g_{ij}(s_i, s_j)$ and $g_{ji}(s_j, s_i)$ represent *the same cardinal scale*.

3 Bilateral Potentials and Anti-Potentials

Our first step is to identify revealed preference conditions implying that the strategic interaction between two players is entirely captured via a single function of the two players' actions. If we can translate and rescale utilities so that $g_{ij}(s_i, s_j) = g_{ji}(s_j, s_i)$, then we call the common function a **bilateral potential**. If instead we get $g_{ij}(s_i, s_j) = -g_{ji}(s_j, s_i)$, then g_{ij} is a **bilateral anti-potential**.

Before we introduce the key condition, we highlight its direct implications on g_{ij} and g_{ji} to aid interpretation. The essence of our characterization is a reciprocal relationship between cross-differences. Take three actions $s_i, s'_i, s''_i \in S_i$ for player i and two actions $s_j, s'_j \in S_j$ for player j , and suppose that

$$\begin{aligned} g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) - (g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j)) \\ = g_{ij}(s''_i, s'_j) - g_{ij}(s'_i, s'_j) - (g_{ij}(s''_i, s_j) - g_{ij}(s'_i, s_j)). \end{aligned} \quad (3)$$

This means that, when opponent j switches from s_j to s'_j , this change has the same impact on i 's choice between s_i and s'_i as it does on i 's choice between s'_i and s''_i . Whenever this occurs, we require that also

$$\begin{aligned} g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i) - (g_{ji}(s'_j, s_i) - g_{ji}(s_j, s_i)) \\ = g_{ji}(s'_j, s''_i) - g_{ji}(s_j, s''_i) - (g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i)). \end{aligned} \quad (4)$$

Intuitively, when opponent i switches from s_i to s'_i , this change has the same impact on j 's choice between s_j and s'_j as when i switches from s'_i to s''_i .

Figure 1 illustrates, plotting i 's actions on the horizontal axis and j 's actions on the vertical axis. The three vertical lines in red represent changes in player j 's action from s_j to s'_j at each of i 's three actions s_i, s'_i , and s''_i . The four horizontal lines in blue represent changes in player i 's action, either from s_i to s'_i or from s'_i to s''_i , at both of j 's actions s_j and s'_j . Our condition says that, when the cross-differences for player i are equal across the two squares, then the cross-differences for player j are also equal across the two squares. Our theorem shows that, whenever cross-differences exhibit such reciprocity, then

$$g_{ij}(s_i, s_j) = \gamma_{ij}g_{ji}(s_i, s_j) + v_{ij}(s_i) + w_{ij}(s_j)$$

for some constant γ_{ij} and functions v_{ij}, w_{ij} that depend only on s_i and s_j respectively. If $\gamma_{ij} > 0$, we have a bilateral potential, and if $\gamma_{ij} < 0$, we have a bilateral anti-potential.

A revealed preference restriction that implies these reciprocal cross-differences is the following **reciprocal prism condition**.

Definition 1. *The preferences of players i and j satisfy the **reciprocal prism condition** if, whenever we have the indifferences*

$$(s_i, s_j, s_{-ij}) \simeq_i (s'_i, s_j, r_{-ij}), \quad (s_i, s'_j, s_{-ij}) \simeq_i (s'_i, s'_j, t_{-ij}),$$

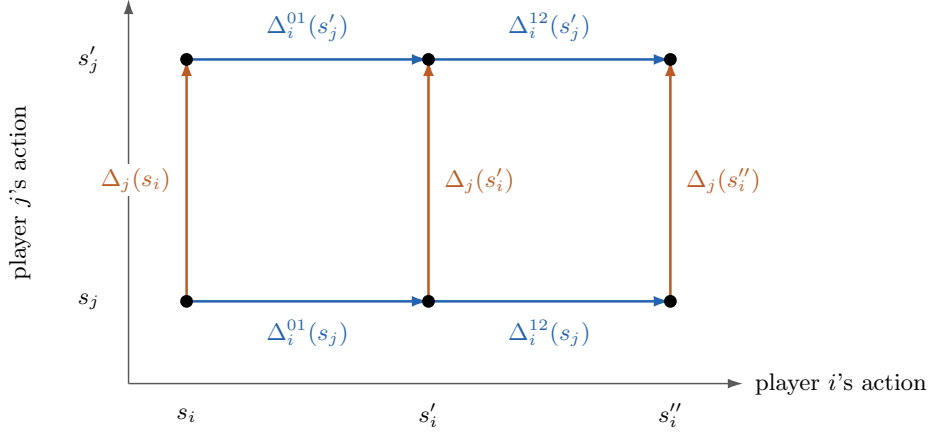
$$(s'_i, s_j, t_{-ij}) \simeq_i (s''_i, s_j, q_{-ij}), \quad \text{and} \quad (s'_i, s'_j, r_{-ij}) \simeq_i (s''_i, s'_j, q_{-ij})$$

for player i , then given any alternative opponent profiles $\tilde{s}_{-ij}, \tilde{t}_{-ij}, \tilde{r}_{-ij}$, and $\tilde{q}_{-ij}$ satisfying

$$(s_j, s_i, \tilde{t}_{-ij}) \simeq_j (s'_j, s_i, \tilde{s}_{-ij}), \quad (s_j, s'_i, \tilde{r}_{-ij}) \simeq_j (s'_j, s'_i, \tilde{s}_{-ij}), \quad \text{and} \quad (s_j, s''_i, \tilde{r}_{-ij}) \simeq_j (s'_j, s''_i, \tilde{q}_{-ij}),$$

we also have

$$(s_j, s'_i, \tilde{t}_{-ij}) \simeq_j (s'_j, s'_i, \tilde{q}_{-ij}).$$



$$\begin{aligned} [\Delta_i^{01}(s'_j) - \Delta_i^{01}(s_j)] &= [\Delta_i^{12}(s'_j) - \Delta_i^{12}(s_j)] \\ \implies [\Delta_j(s'_i) - \Delta_j(s_i)] &= [\Delta_j(s''_i) - \Delta_j(s'_i)]. \end{aligned}$$

$$\begin{aligned} \Delta_i^{01}(t) &:= g_{ij}(s'_i, t) - g_{ij}(s_i, t), & \Delta_i^{12}(t) &:= g_{ij}(s''_i, t) - g_{ij}(s'_i, t), \\ \Delta_j(r) &:= g_{ji}(s'_j, r) - g_{ji}(s_j, r). \end{aligned}$$

Figure 1: We require that if $\Delta_i^{01}(s'_j) - \Delta_i^{01}(s_j) = \Delta_i^{12}(s'_j) - \Delta_i^{12}(s_j)$ then we also have $\Delta_j(s'_i) - \Delta_j(s_i) = \Delta_j(s''_i) - \Delta_j(s'_i)$.

The name of our condition derives from a geometric interpretation. We divide the comparisons into two groups: four for player i and four for player j . If there is only one other opponent k , then for each set of comparisons, we can plot the eight corresponding points as the vertices of a prism in three-dimensional space, and each indifference curve connects two vertices. Figure 2 depicts the prism for player i , with indifference curves in blue. The reciprocal prism condition requires that, whenever such a prism exists for player i , then the analogous prism depicted in Figure 3, with indifference curves in red, exists for player j .

Parsing Definition 1 requires some care. Suppose that for both i and j , actions for the other opponents are rich enough that we can always find four profiles ensuring that the first three comparisons hold—this is true if, for instance, the sums $\sum_{k \neq i, j} g_{ik}(s_i, s_k)$ and $\sum_{k \neq i, j} g_{jk}(s_j, s_k)$ are unbounded above (or below) at each s_i and s_j when we vary s_{-ij} . The substance of the definition is its assertion that, if the fourth indifference holds for player i , then the fourth indifference also holds for player j .

To see the relationship between Definition 1 and the cross-differences discussed earlier, apply the additively separable utility representations:

$$u_i(\mathbf{s}) = \sum_{k \neq i} g_{ik}(s_i, s_k), \quad u_j(\mathbf{s}) = \sum_{k \neq j} g_{jk}(s_j, s_k).$$

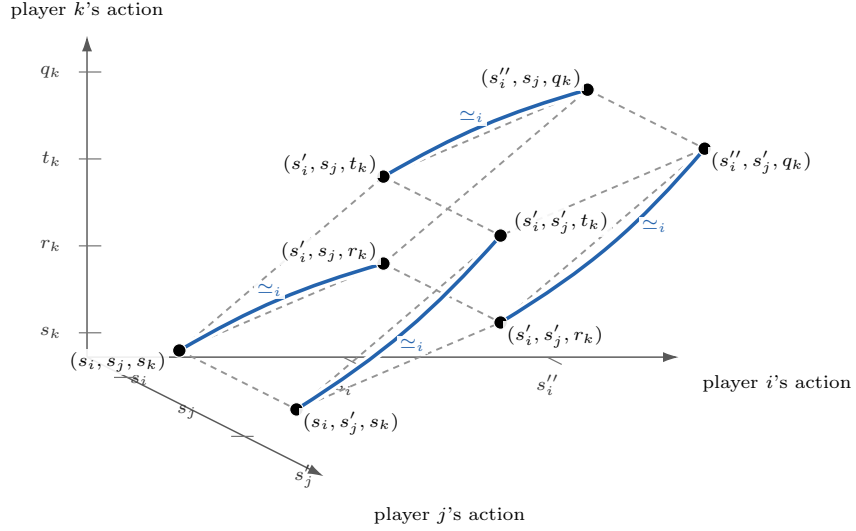


Figure 2: The first prism showing player i 's indifference

Looking at player i , the first two comparisons imply that

$$g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j) = \sum_{k \neq i, j} g_{ik}(s_i, s_k) - g_{ik}(s'_i, r_k), \quad \text{and}$$

$$g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) = \sum_{k \neq i, j} g_{ik}(s_i, s_k) - g_{ik}(s'_i, t_k).$$

Subtracting gives

$$g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) - (g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j)) = \sum_{k \neq i, j} g_{ik}(s'_i, r_k) - g_{ik}(s'_i, t_k).$$

Similarly, the second two comparisons imply

$$g_{ij}(s''_i, s'_j) - g_{ij}(s'_i, s'_j) - (g_{ij}(s''_i, s_j) - g_{ij}(s'_i, s_j)) = \sum_{k \neq i, j} g_{ik}(s'_i, r_k) - g_{ik}(s'_i, t_k),$$

so we have equal cross-differences:

$$\begin{aligned} & g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) - (g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j)) \\ &= g_{ij}(s''_i, s'_j) - g_{ij}(s'_i, s'_j) - (g_{ij}(s''_i, s_j) - g_{ij}(s'_i, s_j)). \end{aligned}$$

An analogous calculation shows that the second four comparisons imply

$$\begin{aligned} & g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i) - (g_{ji}(s'_j, s_i) - g_{ji}(s_i, s_j)) \\ &= g_{ji}(s'_j, s''_i) - g_{ji}(s_j, s''_i) - (g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i)), \end{aligned}$$

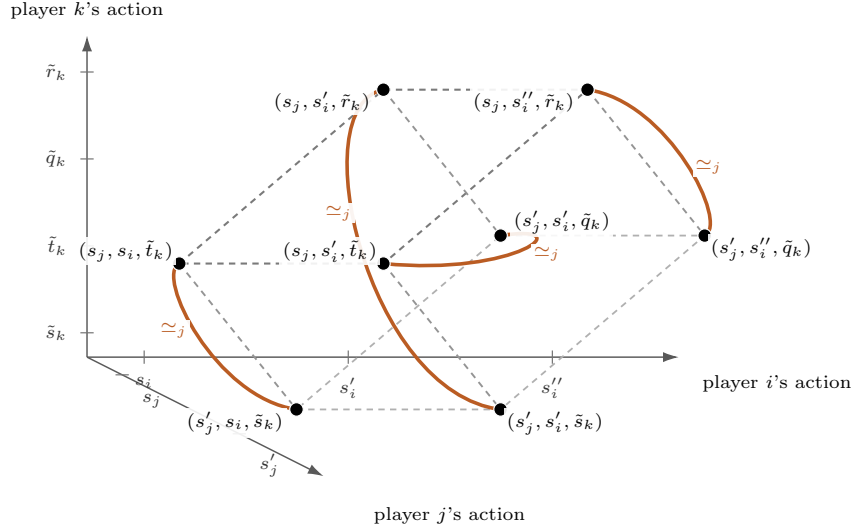


Figure 3: The reciprocal prism showing player j 's indifference

as required.

To ensure that preferences always reveal this cross-difference reciprocity, we do need a richness assumption on the actions of other opponents.

Assumption 1. *For any two players i and j , given any $s_i, s'_i, s''_i \in S_i$ and $s_j, s'_j \in S_j$, we can find $s_{-ij}, r_{-ij}, t_{-ij}, q_{-ij}$ such that the first three indifference for player i in Definition 1 hold. Analogously we can find $\tilde{s}_{-ij}, \tilde{r}_{-ij}, \tilde{t}_{-ij}, \tilde{q}_{-ij}$ such that the first three indifference for player j in Definition 1 hold.*

Assumption 1 directly asserts that we can find profiles for other opponents so that the first three indifference in Definition 1 hold, both for player i and for player j . An easy way to satisfy this assumption is if the sums

$$\sum_{k \neq i, j} g_{ik}(s_i, s_k) \quad \text{and} \quad \sum_{k \neq i, j} g_{jk}(s_j, s_k)$$

are unbounded above (or below) for each $s_i \in S_i$ and $s_j \in S_j$ as we vary s_{-ij} . Under this assumption, the reciprocal prism condition exactly means that equation (3) always implies equation (4).

Lemma 1. *Suppose Assumption 1 holds, and the preferences of players i and j satisfy the reciprocal prism condition. For any $s_i, s'_i, s''_i \in S_i$ and any $s_j, s'_j \in S_j$, having*

$$\begin{aligned} & g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) - (g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j)) \\ &= g_{ij}(s''_i, s'_j) - g_{ij}(s'_i, s'_j) - (g_{ij}(s''_i, s_j) - g_{ij}(s'_i, s_j)). \end{aligned}$$

implies that also

$$\begin{aligned} g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i) - (g_{ji}(s'_j, s_i) - g_{ji}(s_i, s_j)) \\ = g_{ji}(s'_j, s''_i) - g_{ji}(s_j, s''_i) - (g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i)). \end{aligned}$$

Proof. Suppose the first equation holds, relating two cross-differences for player i . Using Assumption 1, we can find four profiles s_{-ij} , r_{-ij} , t_{-ij} , and q_{-ij} such that the first three indifferences in Definition 1 hold. Now, observe that

$$\begin{aligned} \sum_{k \neq i, j} g_{ik}(s'_i, r_k) - g_{ik}(s'_i, t_k) &= g_{ij}(s'_i, s'_j) - g_{ij}(s_i, s'_j) - (g_{ij}(s'_i, s_j) - g_{ij}(s_i, s_j)) \\ &= g_{ij}(s''_i, s'_j) - g_{ij}(s'_i, s'_j) - (g_{ij}(s''_i, s_j) - g_{ij}(s'_i, s_j)) \end{aligned}$$

by assumption. The construction of t_{-ij} and q_{-ij} implies that

$$g_{ij}(s''_i, s_j) - g_{ij}(s'_i, s_j) = \sum_{k \neq i, j} g_{ik}(s'_i, t_k) - g_{ik}(s''_i, q_k),$$

so we must also have

$$g_{ij}(s''_i, s'_j) - g_{ij}(s'_i, s'_j) = \sum_{k \neq i, j} g_{ik}(s'_i, r_k) - g_{ik}(s''_i, q_k),$$

which implies $(s'_i, s'_j, r_{-ij}) \simeq_i (s''_i, s'_j, q_{-ij})$. Applying Assumption 1 again, we can find alternative opponent profiles $\tilde{s}_{-ij}$, $\tilde{t}_{-ij}$, $\tilde{r}_{-ij}$, and $\tilde{q}_{-ij}$ satisfying

$$(s_j, s_i, \tilde{t}_{-ij}) \simeq_j (s'_j, s_i, \tilde{s}_{-ij}), (s_j, s'_i, \tilde{r}_{-ij}) \simeq_j (s'_j, s'_i, \tilde{s}_{-ij}), \text{ and } (s_j, s''_i, \tilde{r}_{-ij}) \simeq_j (s'_j, s''_i, \tilde{q}_{-ij}),$$

and the reciprocal prism condition then gives

$$(s_j, s'_i, \tilde{t}_{-ij}) \simeq_j (s'_j, s'_i, \tilde{q}_{-ij}).$$

A similar calculation now implies that

$$\begin{aligned} g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i) - (g_{ji}(s'_j, s_i) - g_{ji}(s_i, s_j)) \\ = g_{ji}(s'_j, s''_i) - g_{ji}(s_j, s''_i) - (g_{ji}(s'_j, s'_i) - g_{ji}(s_j, s'_i)) \end{aligned}$$

as needed. □

Our main result in this section shows that, given the additively separable utility representation we obtained in part I, the reciprocal prism condition ensures that the corresponding pairwise terms in the two players' utilities are essentially constant multiples of one another.

Theorem 1. *Suppose Assumption 1 holds, and the preferences of players i and j satisfy the reciprocal prism condition. There exists a constant γ_{ij} , and functions $v_{ij}(s_i)$ and $w_{ij}(s_j)$, such that*

$$g_{ij}(s_i, s_j) = \gamma_{ij} g_{ji}(s_j, s_i) + v_{ij}(s_i) + w_{ij}(s_j).$$

Using Lemma 1, we can work directly with the cross-differences to prove Theorem 1. A second technical lemma shows that cross-difference reciprocity implies one function is an affine transformation of the other.

Lemma 2. *Suppose $f(x)$ and $g(x)$ are continuous real-valued functions on a path-connected topological space X . If*

$$f(x) + f(z) = 2f(y) \implies g(x) + g(z) = 2g(y),$$

then $g(x) = af(x) + b$ for some real constants a and b .

Proof. First, observe that if f is constant, the conclusion is immediate. A constant f implies that $f(x) + f(z) = 2f(y)$ for all $x, y, z \in X$, which means $g(x) + g(z) = 2g(y)$ for all $x, y, z \in X$. Taking $y = x$ and simplifying then gives $g(z) = g(x)$ for all $x, z \in X$, so g is constant.

Now suppose $f(x_0) < f(x_1)$, and define

$$a = \frac{g(x_1) - g(x_0)}{f(x_1) - f(x_0)}, \quad b = g(x_0) - af(x_0).$$

The function $D(x) = g(x) - af(x) - b$ satisfies

$$f(x) + f(z) = 2f(y) \implies D(x) + D(z) = 2D(y).$$

Our goal is to show that $D(x)$ is identically zero. Note by construction we already have $D(x_0) = D(x_1) = 0$.

Since X is connected, by the intermediate value theorem we can find a point $x_{1/2}$ such that $f(x_0) + f(x_1) = 2f(x_{1/2})$. It follows that $g(x_0) + g(x_1) = 2g(x_{1/2})$, and

$$D(x_{1/2}) = \frac{D(x_0) + D(x_1)}{2} = 0$$

as needed. Proceeding inductively, we can construct analogous points of the form $x_{k/2^n}$ for all $n \in \mathbb{N}$ and $0 < k < 2^n$, with $D(x_{k/2^n}) = 0$ at all such points. For any other point x with $f(x) \in [f(x_0), f(x_1)]$, we can find a sequence of points $y_1, y_2, \dots$ along a path between x_0 and x_1 of the form $x_{k/2^n}$ such that $f(y_m) \rightarrow f(x)$. As the inverse image of the path from x_0 to x_1 is compact, a subsequence of these points converges to some point y with $f(y) = f(x)$. Since $D(y_m) = 0$ for each m , continuity implies $D(y) = D(x) = 0$.

Now let $I \subseteq \mathbb{R}$ the set of all points such that $D(x) = 0$ whenever $f(x) \in I$. Following a similar argument as in the last paragraph, the set I must be closed. If I does not contain all of $f(X)$, we can find a point z with $f(z) \notin I$ —without loss, assume $f(z) > f(x_0)$. We can repeat our earlier argument, replacing x_1 with z , to show that $g(x) = \tilde{a}f(x) + \tilde{b}$ for some constants $\tilde{a}$ and $\tilde{b}$ whenever $f(x) \in [x_0, z]$. Since this interval overlaps with $[x_0, x_1]$, it must be that $\tilde{a} = a$ and $\tilde{b} = b$. We conclude that $f(z) \in I$, and $I = f(X)$, completing the proof. $\square$

With this result in hand, we now prove Theorem 1.

Proof of Theorem 1. Define

$$\Delta_{ij}(s_i, s'_j, s_j) = g_{ij}(s_i, s'_j) - g_{ij}(s_i, s_j), \quad \text{and}$$

$$\Delta'_{ij}(s_i, s'_j, s_j) = g_{ji}(s'_j, s_i) - g_{ji}(s_j, s_i).$$

From Lemma 1, the reciprocal prism condition together with our richness assumption means that whenever

$$\Delta_{ij}(s''_i, s'_j, s_j) - \Delta_{ij}(s'_i, s'_j, s_j) = \Delta_{ij}(s'_i, s'_j, s_j) - \Delta_{ij}(s_i, s'_j, s_j),$$

we also have

$$\Delta'_{ij}(s''_i, s'_j, s_j) - \Delta'_{ij}(s'_i, s'_j, s_j) = \Delta'_{ij}(s'_i, s'_j, s_j) - \Delta'_{ij}(s_i, s'_j, s_j).$$

Applying Lemma 2, for every s_i , s_j , and s'_j we have

$$\Delta'_{ij}(s_i, s'_j, s_j) = a_i(s'_j, s_j) \Delta_{ij}(s_i, s'_j, s_j) + b_i(s'_j, s_j)$$

for some functions a_i and b_i . This implies

$$\begin{aligned} \Delta_{ji}(s'_j, s'_i, s_i) - \Delta_{ji}(s_j, s'_i, s_i) &= \Delta'_{ij}(s'_i, s'_j, s_j) - \Delta'_{ij}(s_i, s'_j, s_j) \\ &= a_i(s'_j, s_j) (\Delta_{ij}(s'_i, s'_j, s_j) - \Delta_{ij}(s_i, s'_j, s_j)) \end{aligned}$$

Similarly, we have

$$\Delta'_{ji}(s_j, s'_i, s_i) = a_j(s'_i, s_i) \Delta_{ji}(s_j, s'_i, s_i) + b_j(s'_i, s_i),$$

implying

$$\Delta_{ij}(s'_i, s'_j, s_j) - \Delta_{ij}(s_i, s'_j, s_j) = a_j(s'_i, s_i) (\Delta_{ji}(s'_j, s'_i, s_i) - \Delta_{ji}(s_j, s'_i, s_i)).$$

Since a_i is a function only of s'_j and s_j , and a_j is a function only of s'_i and s_i , this implies that $a_i = \frac{1}{a_j}$ is constant, and the conclusion follows. $\square$

Theorem 1 relates the bilateral payoff terms for players i and j , but on its own this is not enough for a bilateral potential because the result says nothing about the sign of the multiplicative constant γ_{ij} . Moreover, the reciprocal prism condition as stated is a one way implication. If it holds for i and j , but not for j and i , then $\gamma_{ij} = 0$ in the Theorem. If it holds in both directions, then we can identify the sign of γ_{ij} through revealed preferences. Suppose we find $s_i, s'_i, s_j, s'_j, s_{-ij}, s'_{-ij}$ such that

$$(s'_i, s_j, s'_{-ij}) \preceq_i (s_i, s_j, s_{-ij}), \quad (s'_i, s'_j, s'_{-ij}) \succeq_i (s_i, s'_j, s_{-ij})$$

with at least one strict preference. If we then find t_{-ij}, t'_{-ij} such that

$$(s'_j, s_i, t'_{-ij}) \simeq_j (s_j, s_i, t_{-ij}),$$

then γ_{ij} is positive (negative) if and only if

$$(s'_j, s'_i, t'_{-ij}) \succ_j (\prec_j) (s_j, s'_i, t_{-ij}).$$

In words, when γ_{ij} is positive, it must be that j switching from s_j to s'_j makes s'_i more attractive relative to s_i if and only if i switching from s_i to s'_i makes s'_j more attractive relative to s_j .

As a straightforward Corollary to Theorem 1, we have the following canonical representation of player utilities when the reciprocal prism condition holds for all pairs of players.

Corollary 1. *Suppose Assumption 1 holds, and preferences satisfy the reciprocal prism condition for each pair of players. For each player i , we can represent her preferences using a utility of the form*

$$u_i(\mathbf{s}) = v_i(s_i) + \sum_{j \neq i} (\gamma_{ij} g_{ij}(s_i, s_j) + w_{ij}(s_j)) \quad (5)$$

for some functions $v_i, \{g_{ij}\}, \{w_{ij}\}$ and constant weights $\{\gamma_{ij}\}$ such that $g_{ij}(s_i, s_j) = g_{ji}(s_j, s_i)$ for every $i, j \in N$, $s_i \in S_i$, and $s_j \in S_j$.

Going forward, we assume we can always represent preferences via (5), and we write Γ for the corresponding matrix of weights with zeros on the diagonal.

4 Network Potential Games

To have a potential for the entire game, we clearly need all entries of Γ to be non-negative, and we need $\gamma_{ij} > 0$ if and only if $\gamma_{ji} > 0$. Given this, each pair of players i and j is playing a common interest game, holding fixed the actions of other players. This might seem like enough to get a potential for the entire game, but issues can still arise as our next example shows. For the rest of this section, we assume that $\gamma_{ij} \geq 0$ for each pair ij , and $\gamma_{ij} > 0 \iff \gamma_{ji} > 0$. We write G for the **interaction network**, the undirected graph that contains link ij if $\gamma_{ij} > 0$.

Suppose there are n players, each player i has action set $S_i = [-1, 1]$, and payoffs are

$$u_i(\mathbf{s}) = s_i(s_{i-1} + 2s_{i+1}) - \frac{s_i^2}{2}$$

in which we take indices modulo n . Every pair of players i and j has the bilateral potential $s_i s_j$, with $\gamma_{ij} > 0$ if and only if i and j are adjacent in the index order. Nevertheless, this is not a potential game because an improvement cycle exists. Consider the profile in which player i takes action 1 and all other players take action -1 . Player $i - 1$ can do better by switching to action 1, earning $\frac{1}{2}$ instead of $-\frac{3}{2}$. Subsequently, player i can do better by

switching to action -1 , again earning $\frac{1}{2}$ instead of $-\frac{3}{2}$. By repeating this pair of moves for players $i-2$ and $i-1$, then $i-3$ and $i-2$, and so on, we cycle back to the original profile.

Note that a shorter cycle is not possible because if we fix any one player's action, the game among the remaining $n-1$ players is a potential game. Fixing s_i , an exact potential for the game among players $N \setminus \{i\}$ is:

$$\Phi(s) = \sum_{j=1}^{n-1} 2^{j-1} \left(s_{i+j} s_{i+j-1} - \frac{s_{i+j}^2}{2} \right)$$

in which we again take player indices modulo n .

A key feature of this example is that the interaction network G contains a long cycle with no chord—i.e., no path to cut it short. It turns out that such cycles are precisely what makes it difficult to check whether we can simultaneously scale bilateral potentials to obtain a single potential for the entire game. Our main result in this section shows that a bound on the length of chordless cycles in G translates directly into a bound on the length of potential improvement cycles we must check.

Definition 2. A **cycle** C in G is a sequence of vertices $i_1, i_2, \dots, i_\ell$ such that $i_{k-1}i_k \in G$ for each $k = 1, 2, \dots, \ell$, where we identify $i_\ell = i_0$. A **chord** for the cycle C is a link $i_t i_s \in G$, such that i_t and i_s appear in C , but $|t-s| \geq 2$, where the difference is taken modulo ℓ . A cycle C is **chordless** if it has no chord in G .

Mathematically, whether we can rescale utilities so that $g_{ij}(s_i, s_j) = g_{ji}(s_j, s_i)$ for all pairs simultaneously is equivalent to finding a diagonal matrix Λ , with strictly positive entries, such that $\Lambda\Gamma$ is symmetric. We first show that this is equivalent to having

$$\pi(C) := \prod_{i=1}^{\ell} \frac{\gamma_{i_{k-1}i_k}}{\gamma_{i_k i_{k-1}}} = 1 \tag{6}$$

for any cycle C in G —as always, indices in the product should be read modulo ℓ . While checking the above condition may involve checking very long cycles, it turns out that checking (6) only for chordless cycles is sufficient.

Theorem 2. *The following three conditions are equivalent:*

- (a) *There exists a strictly positive diagonal matrix Λ such that $\Lambda\Gamma$ is symmetric*
- (b) *We have $\pi(C) = 1$ for every cycle C in the interaction graph G*
- (c) *We have $\pi(C) = 1$ for every chordless cycle C in the interaction graph G*

Proof. We first show that (a) implies (b). If (a) holds, we have

$$\lambda_i \gamma_{ij} = \lambda_j \gamma_{ji}$$

for every pair ij . For any cycle $C = (i_0, i_1, \dots, i_\ell)$, we therefore have

$$\pi(C) = \prod_{k=1}^{\ell} \frac{\gamma_{i_{k-1}i_k}}{\gamma_{i_k i_{k-1}}} = \prod_{k=1}^{\ell} \frac{\lambda_{i_k}}{\lambda_{i_{k-1}}} = 1$$

since $i_0 = i_\ell$.

Going the other direction, supposing that $\pi(C) = 1$ for every cycle, we construct a constant λ_i for each player i . Fix a spanning tree T in the interaction graph, and normalize $\lambda_1 = 1$. For every other player i , there is a unique path $1 = i_1, i_2, \dots, i_\ell = i$ from 1 to i in T , and we can use this path to define λ_i :

$$\lambda_i = \prod_{k=2}^{\ell} \frac{\gamma_{i_{k-1}i_k}}{\gamma_{i_k i_{k-1}}}.$$

We now verify that

$$\lambda_i \gamma_{ij} = \lambda_j \gamma_{ji}$$

for every $ij \in G$. First, if ij is in the spanning tree T , then without loss take i to be closer to 1 in T , so ij is the last link on the unique path from 1 to j . By construction, we have

$$\lambda_j = \lambda_i \cdot \frac{\gamma_{ij}}{\gamma_{ji}}$$

as required. If $ij \in G$ but $ij \notin T$, then there is a unique path $i = i_1, i_2, \dots, i_\ell = j$ from i to j in T , and adding link ij completes a cycle C on this set of players. By construction, we have

$$\frac{\lambda_j}{\lambda_i} = \prod_{k=2}^{\ell} \frac{\gamma_{i_{k-1}i_k}}{\gamma_{i_k i_{k-1}}},$$

and we then have

$$1 = \pi(C) = \prod_{k=1}^{\ell} \frac{\gamma_{i_{k-1}i_k}}{\gamma_{i_k i_{k-1}}} = \frac{\lambda_j \gamma_{ji}}{\lambda_i \gamma_{ij}}$$

as required.

Condition (b) obviously implies (c), so we only argue the reverse implication for this part. We induct on the length of a cycle—if $\pi(C) = 1$ for every chordless cycle, then we clearly have $\pi(C) = 1$ for every cycle of length at most 3 as every cycle of length at most 3 is chordless. Suppose now that $\pi(C) = 1$ for every chordless cycle and every cycle of length less than ℓ —we show that $\pi(C) = 1$ for every cycle of length ℓ .

Let $C = i_1, i_2, \dots, i_\ell$ be a cycle of length ℓ . If it is chordless, we are done. If it has a chord $i_t i_s$ with $t < s$, we can divide C into two cycles:

$$C_1 = i_1, i_2, \dots, i_t, i_s, i_{s+1}, \dots, i_\ell, \quad C_2 = i_t, i_{t+1}, \dots, i_s.$$

Note that the cycles C_1 and C_2 share exactly one edge, connecting i_t to i_s , but it appears in opposite orientations. Note also that both cycles have length strictly less than ℓ . Hence, we have

$$1 = \pi(C_1)\pi(C_2) = \pi(C) \cdot \frac{\gamma_{i_t i_s}}{\gamma_{i_s i_t}} \cdot \frac{\gamma_{i_s i_t}}{\gamma_{i_t i_s}} = \pi(C)$$

as required. $\square$

Theorem 2 builds on earlier work that examines conditions under which best response dynamics converge in a standard linear-quadratic network game. Bayer et al. (2023) show the equivalence of (a) and (b) for this restricted class of games. We extend this equivalence to all network games that satisfy the reciprocal prism condition, and the further equivalence to (c) narrows what we need to check in order to ensure that a potential exists. An immediate Corollary of Theorem 2 is that, if the graph is chordal—meaning all cycles of length 4 or greater have a chord—then it is enough to check that $\pi(C) = 1$ for every cycle of length 3. We expect this to be a typical case as real networks are highly clustered, which implies the existence of many connections between neighbors of neighbors, and natural incentive structures for network formation lead to stable graphs that are chordal (Sadler, 2026).

Corollary 2. *If G is chordal, and $\pi(C) = 1$ for every cycle of length 3, then the network game has a potential.*

5 Discussion

Given a game with separable preferences across opponents, the reciprocal prism condition provides a revealed preference test of whether two players’ utilities share a compatible cardinal scale. If additionally the two players’ interactions exhibit strategic complements of a sort—player i moving from s_i to s'_i favors s'_j over s_j and vice versa—then we can rescale their utilities so that their interaction is one of pure common interest. Whether we can do this simultaneously for all pairs of players turns on whether utility scales are consistent along chordless cycles in the interaction network. In a prominent class of networks, we need only examine cycles of length 3.

As stated, our results require a game to have at least three players, and every player to have at least two with whom she meaningfully interacts. This is crucial even to state the key reciprocal prism condition because for any link ij , player i (j) must have another opponent whose actions can offset those of j (i). However, there is nothing special about having an opponent as the offsetting force, and if two players have preferences over some other set of outcomes, preferences that are independent of outcomes in the game, this can serve the same role, allowing us to recover a common cardinal scale. Our findings may therefore find use in simpler settings like two-player games as well.

Several directions call for further work. While we implicitly characterize games that feature bilateral conflict, we say little else about this potentially interesting class. While these are not potential games, their special structure may prove helpful in other ways. In part I, we identified a number of further revealed preference conditions—constant rate of substitution,

midpoint indifference—with implications for the specific form that the terms g_{ij} must take. Combining these insights with our Theorem 1 can offer a more detailed taxonomy of utility functions and their substantive implications. We believe such a taxonomy can provide a useful starting point for a robust analysis of equilibrium outcomes in network games.

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